Using Monte Carlo Simulation with Oracle© Crystal Ball to Teach Business Students Hypothesis Testing Concepts and Type I Error

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ABSTRACT

This article explains a basic Monte Carlo simulation workshop applied to teaching fundamental hypothesis testing concepts. The workshop can be conducted in a classroom or lab where students have access to a Monte Carlo simulation tool such as Oracle© Crystal Ball. To our knowledge, this is the first time a workshop has been conducted and documented utilizing Monte Carlo analysis to teach students basic ideas regarding hypothesis testing and type I error theory. The results of a post-workshop survey showed that the majority of students found the workshop to be effective and useful. Furthermore, for an end-of-semester project where students could choose any business statistics topic, the majority picked hypothesis testing and demonstrated competence regarding the theory and application.

Keywords: Teaching Hypothesis Testing, Monte Carlo Simulation, Business Statistics, Type I Error

INTRODUCTION

A foundational topic in most introductory business statistics courses is hypothesis testing, a methodology in which sample data is collected and analyzed to determine whether a claim of interest can be supported. This “claim of interest” is also called the alternate hypothesis, H_1, or the research hypothesis. Students should easily be aware of potential problems that could occur when organizations make claims that turn out to be untrue (lawsuits, liability, loss of goodwill, and the like). In more technical terms, this problem (referred to as committing a type I error) involves rejecting the null hypothesis and supporting the alternative or research hypothesis when the null is true. In performing business statistical test, we certainly want to avoid such issues. Many analysts commonly set the likelihood of committing this kind of mistake at 0.05 or 5% (Nuzzo, 2014).

This manuscript is the second part of a series in using Monte Carlo analysis to demonstrate foundational business statistics concepts (Weltman, 2015). We have found aspects of several foundational concepts (sampling distributions, confidence intervals, degrees of freedom) in a basic business statistics course to be somewhat difficult for students to understand. Being able to use tools that demonstrate and reinforce “abstract theory” has proven to be helpful to students in our experience. Hypothesis testing concepts are certainly no exception to these observations of conceptual difficulties (Holland, 2012; Dambolena, I., et. al., 2009; Sotos, et. al., 2009; Aguinis & Branstetter, 2007; Loosen, 1997).

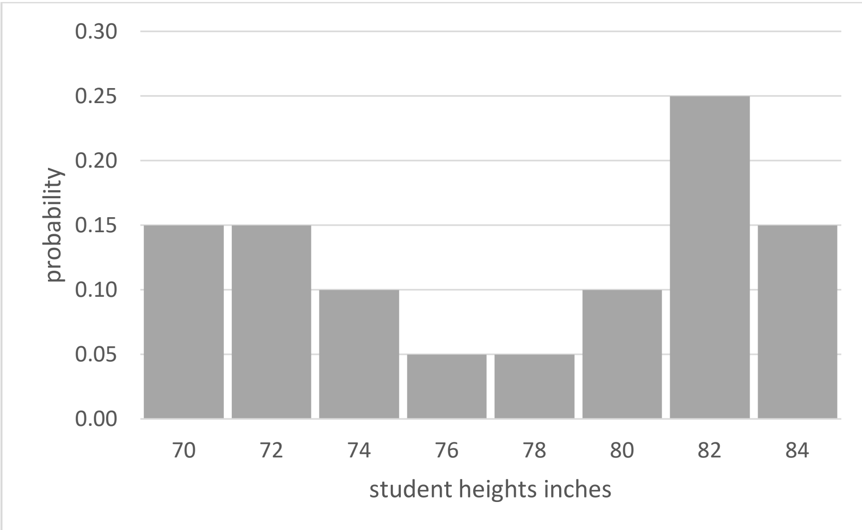
The use of Monte Carlo simulation with Oracle Crystal Ball has shown to be an enjoyable and effective way to reinforce theory. Moreover, many students report continued use of the tool in subsequent course projects, internships, and full-time positions. Introducing Monte Carlo analysis early in a student’s college career (a typical Business Statistics course is sophomore year) thus fulfills many productive purposes.

Monte Carlo simulation is an iterative mathematical technique that is used to approximate the likelihood of outcomes by running many thousands of “trial” scenarios. The technique numerically quantifies and graphically depicts potential results (along with their associated likelihoods) based on provided uncertain inputs. Kersten (1983) states that by performing simulations in introductory statistics courses, non-mathematically oriented students can have an inductive learning experience in a time-efficient manner. Kennedy (1998) stresses the importance of sampling distributions and calls for instructors to spend a considerable amount of time on this topic with Monte Carlo simulation. Tools are available for instructors to answer these calls more easily as Monte Carlo applications have evolved greatly over the past several years. Currently, the technique is widely used in numerous business applications (see “Who Uses Monte Carlo Simulation” at www.microsoft.com; “When to Use Monte Carlo Simulation” at www.ibm.com, or Winston, 2004; Kwak, & Ingall, 2007; Engle, Granger, & Hallman, 1989). Valle & Norvell (2013) use Oracle Crystal Ball in the area of forecasting demonstrations and cite numerous instances of businesses using Monte Carlo simulation analysis. In the present research, the technique is used in a hands-on environment to provide students an experience in core business statistics concepts: hypothesis testing, type I error, and the central limit theorem.

THE WORKSHOP

In the scenario employed in our workshop, students download an Excel file that contains a worksheet that shows the population distribution of student heights (Figure 1) at a hypothetical large university of 20,000 students.

Figure 1: The Population Distribution



The mean and standard deviation of heights for all students are μ=77.50 and σ=5.13 inches, respectively. The students are asked to set up a research hypothesis that the mean height for all the students is under 77.5 inches:

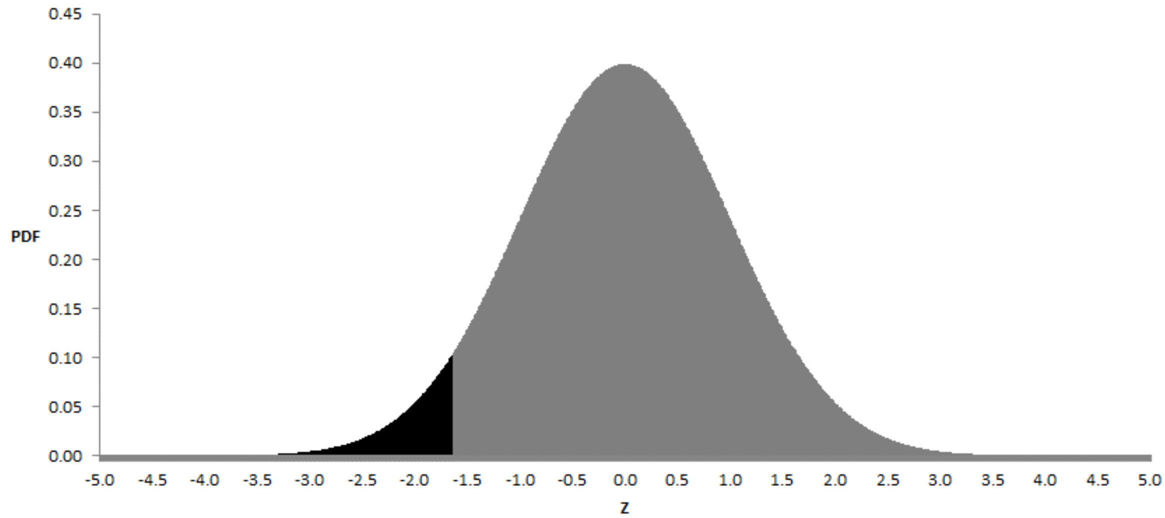
$$H_0: \mu \geq 77.5$$

$$H_1: \mu < 77.5$$

Clearly a mistake would be made if an analyst were to arrive at this conclusion since the true mean height is 77.5 inches. This kind of mistake is called a type 1 error. Using the typical 5% (α=0.05) level of significance to control for this error, we would expect that only about 5% of the time would we obtain sample means where our research hypothesis is supported in error, i.e. concluding H_1 is true when it is not. Next, the students are asked to use statistical theory to determine the sample mean height from a sample of size 30 that would cause this kind of mistake, using a 0.05 level of significance where σ is known and is 5.13 inches, the standard deviation of the population distribution of Figure 1.

When the population standard deviation is known the Z distribution is the appropriate sampling distribution to determine this value. Figure 2 below shows the sampling distribution with the critical value for the hypothesis test being -1.645.

Figure 2: The Sampling Distribution



The central limit theorem expresses that the sampling distribution will be approximately normally distributed with;

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Mathematically, the result is determined by solving the formula below for $\bar{x}$.

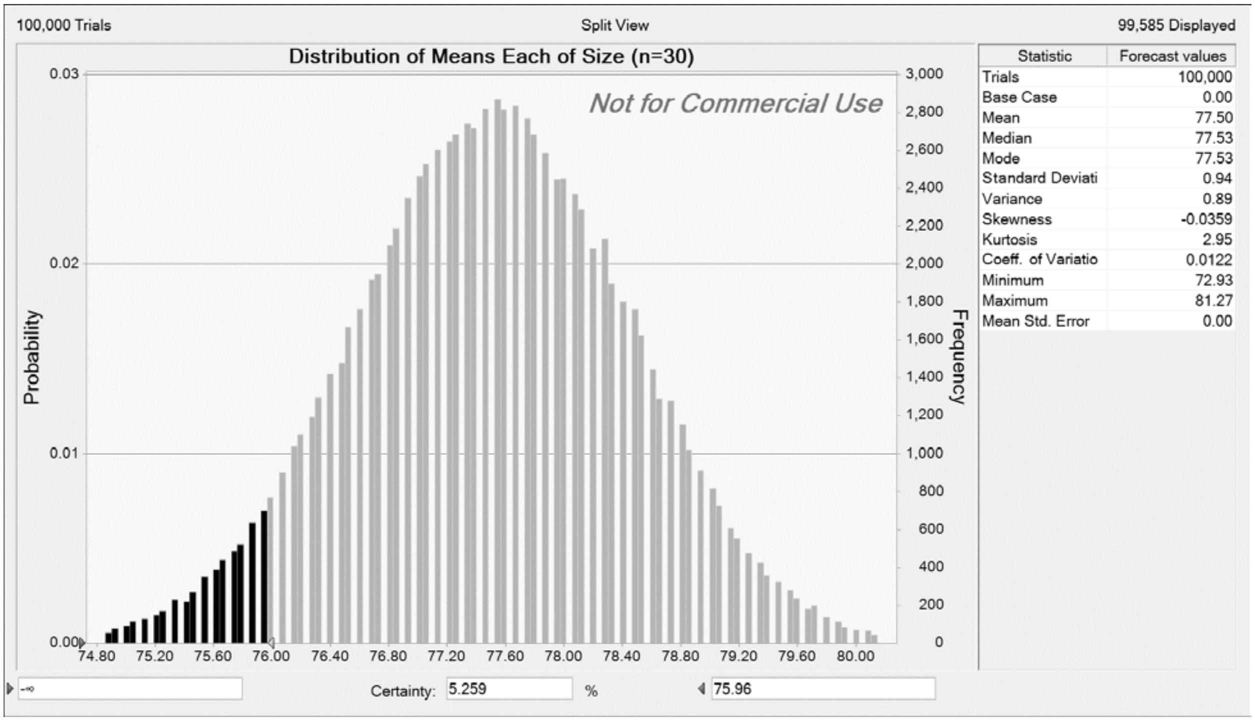
$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$-1.645 = \frac{\bar{x} - 77.5}{5.13 / \sqrt{30}}$$

$$\bar{x} = 75.96$$

Thus, if a random sample of 30 students is taken from the population specified and yields a sample mean of 75.96 inches (or smaller), one would incorrectly conclude the true mean height is less than 77.5 inches, when it is not. Now students are asked to see if Monte Carlo simulation supports their result. Specifically, students are asked to use Monte Carlo simulation to generate 100,000 samples, each sample of size 30, from the specified population distribution (Figure 1) and generate a plot of those sample means. Following the generation of this plot, use the application to specify the sample mean height values that would cause a type 1 error, and approximate the percentage of samples in which these results would occur. Figure 3 below shows an example Monte Carlo simulation plot with the shaded area illustrating obtained sample means that cause a type 1 error. All other sample means would result in the analyst making a correct conclusion of not supporting H_1.

Figure 3: Example Monte Carlo Simulation Estimate of the Sampling Distribution



The Monte Carlo simulation trials approximately match the statistical theory. The distribution is normal in shape even though the population distribution is far from normal. In this set of 100,000 samples, 5.259% of the sample means obtained were less than 75.96 inches. Approximately 95% of the time the sample means were greater than this amount, so we would only make an incorrect conclusion about 5% of the time (sample means obtained that were in the dark-shaded left portion of Figure 3), a match to hypothesis testing statistical theory. Further, notice that the mean of the Monte Carlo estimated sampling distribution is 77.5 inches and the standard deviation of the sampling distribution is 0.94 inches, also matching statistical theory from the central limit theorem, which states that the mean of sampling distribution is equal to the mean of the population and the standard deviation of the sampling distribution is equal to the population standard deviation divided by the square root of the sample size.

$$\mu_{\bar{x}} = \mu = 77.5$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5.13}{\sqrt{30}} = 0.94$$

SUMMARY

We have used Monte Carlo simulation to demonstrate the validity of hypothesis testing statistical theory. Further, we have developed a workshop that enables students to gain "hands-on" understanding of concepts by using a popular general-purpose, commercial software tool. Students work with the tool to understand how a type I error occurs and how often it occurs in sample from a population of interest. Through this hands-on approach, students seem to assimilate hypothesis testing concepts well as was demonstrated in a final class project involving three course sections. In our post workshop survey (74 Business Statistics student participants), 95% of the students said that they now have a basic understanding of Monte Carlo simulation analysis, and 74.3% of the students agreed that this kind of workshop was an effective way to learn concepts in hypothesis testing. 97.3% of the students achieved success on a key hypothesis testing concept question following the workshop. With easy to use Monte Carlo simulation tools readily available, we continue to explore powerful ways of deployment in which students can actively experience business statistical theory.

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David Weltmam holds a Ph.D. in Business Statistics and Master of Science Degrees in Operations Research and in Information Systems. He has been teaching courses in Supply Chain Management, Business Statistics, and Operations Management at the EMBA, MBA, and undergraduate levels both in the U.S. and abroad for over 10 years. Prior to his academic career, David has over 15 year years of business experience with IBM Corporation in Sales and Consulting Services primarily working with organizations in the distribution industry.